Mathematics and AI

Laurent LAFFORGUE

(Lagrange Centre, Huawei Technologies France)

Beijing Foreign Studies University

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Basic math principles of AI in its present form

- Any computing system, such as an AI system,
  necessarily implements ideas of mathematical nature
  which can be fully explicit or partly implicit.

  → Human beings are theoretical:
  they may try to implement and realize
  what they already concieve
  in the form or more or less elaborated views, i.e. "theories".

- AI in its present form
  is based on elementary mathematical principles:

  → The idea (which defines "statistics")
  that there are "laws of large numbers".

  → The idea to approximate
  finite discrete (but big) collections of data
  by continuous functions.

  → The idea to construct approximating functions
  no more by "superposition" (i.e. linear combinations")
  but by "composition".

  → The idea of "incremental improvements"
  (already at the basis of "Newton's method").

Utility for mathematicians of AI in its present form

1) Code writing

AI systems are already very good at code writing,
for instance in "Python".

→ Using AI, it is nowadays possible
to write complex codes even in computer languages that
we did not study.

→ But it is necessary to imagine
architectures of programs, decomposed in steps,
so that an automatic system
can propose a code (and an explanation) for each step.

→ This may be very useful
for mathematicians
who want to use computers to make tries
. . .
or also for scholars
who need elaborate websites!

Utility for mathematicians of AI in its present form

2) Assistance in routine work

An important part of math writing

consists in routine computations,

routine argumentations,

routine checking,

...

→ AI systems are already efficient enough to replace us in routine work, at least partly.

Warning:

Maybe routine work is needed

to become and remain familiar

with mathematical beings

··· just as technical exercises

are necessary for professional musicians.

Utility for mathematicians of AI in its present form

3) Automatic translations in different languages

AI is already advanced enough
to allow automatic translations
of math texts (including Tex files)
in different languages.

→ This makes it possible
to combine language diversity
and international communication.

→ This possibility is very important
as every language
represents a specific way of thinking.

→ Mathematics and more generally sciences
are more rich and creative
if they are practiced in a diversity of languages.

Utility for mathematicians of AI

4) Looking for relevant references and related work

AI systems may help mathematicians and scientists
to find in the ever-increasing "Babel library"
of available articles and books
references which are relevant
for their work.

→ More advanced forms of AI
could possibly help mathematicians
who work on some problems
in a particular domain
to relate these problems
to already existing articles or books
in other domains.

Problems of AI and mathematics

Challenges of AI:

- In their present forms, AI systems need huge data for their parametrization.
  → For instance, all available writings are already used for the parametrization of LLM's.
  → This means that an upper bound has already been reached.
- Computations of AI systems parameters and even ordinary functioning of these systems need too much energy.
- We would want more adaptable AI systems, in the sense that they could perform some tasks on the basis of training for other tasks.

Challenges for mathematicians:

- Present AI systems are "black boxes".
- Mathematicians aim to understand.
- Understanding would bring more efficiency, robustness, reliability, adaptability.

Mathematics for the future development of AI

- We need to integrate to AI systems

  - logic (which formalizes reasoning and can be checked with absolute rigor),
  - topology (i.e. the science of geometric forms considered in a qualitative way, which brings understanding and corresponds to natural human mind processes).

- We need learning processes to elaborate

  - description languages (of formal nature) for natural data we want to process, including images or texts,
  - spaces classifying different types of data, including "spaces of images" or "spaces of texts".

Mathematics for the future development of AI

- We need a mathematical theory
  of "semantic information"
  and of the duality between

  $\begin{cases} - & \text{"syntax" (i.e. formal languages),} \\ - & \text{"semantics" (i.e. meaning).} \end{cases}$

  Such a theory
  should inspire computation processes.
- We need a mathematical modelling of

  "meta-learning"

  i.e. processes for considering and processing
  data at different levels of abstraction.

$\Rightarrow$ The mathematical theory of "toposes",
discovered and developed by A. Grothendieck
is a most inspiring candidate to fill these needs.