On Grothendieck's vision

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"Vision"

- The word "vision" appears
  - more than 400 times in "Récoltes et Semailles" ("Harvests and Sowings"),
  - more that 100 times in "La Clef des Songes" ("The Key to Dreams").

- "Vision" is a French word, which originated from Latin ("visio").

- In French, the direct meaning of this word is concrete:
  - (1) Perception by the eye of light, colours, shapes.
  - (2) Action of seeing, the fact of seeing.
  - (3) What meets the eye.

- The figurative meaning is derived from the concrete:
  - (4) Action, the act of perceiving or imagining a concrete or abstract reality.
  - (5) Image, mental representation of a reality.

Seeing versus reasoning

"Dieu, j'en suis persuadé (et même quand Il fait des maths),
ne raisonne jamais mais toujours il voit
(y compris les relations que nous appelons des raisons,
et que nous enchaînons en des raisonnements)."

("La Clef des Songes")

"God, I am convinced (even when He does maths), <u>never reasons</u> but <u>always sees</u> (including the <u>relationships</u> that we call <u>reasons</u>, and that we link together in <u>reasonings</u>)."

- → As <u>human beings</u> with <u>limited abilities</u>, we <u>make reasonings</u>. If we were <u>like God</u>, <u>we would just see</u>.
- → This is the <u>ideal of Grothendieck</u> in mathematics: <u>replace</u> as much as possible <u>reasoning by seeing</u>.

The necessary and sufficient condition for seeing

"Les choses qui sollicitent l'attention sont innombrables
et il est impossible de suivre jusqu'au bout l'appel de chacune !
[. . .]
Ce qui fait la qualité de l'inventivité et de l'imagination du chercheur,
c'est la qualité de son attention à l'écoute de la voix des choses."

("Récoltes et Semailles")

"There are countless things that demand attention, and
it is impossible to follow every single call of each one to the end!
[. . .]
What makes a researcher's inventiveness and imagination so valuable
is the quality of his attention, listening to the voice of things."

- → What is needed is attention.
- → In order to see,
  what is required is listening!
- → Things have a voice
  and they make calls which we can follow if we listen to them.

Remarks:

- These key ideas are deeply religious.
- "Attention" is also central for philosopher Simone Weil (1909-1943),
  the sister of mathematician André Weil (1906-1998).

Seeing from new points of view

"Plus encore que vers la découverte de questions, de notions et d'énoncés nouveaux, c'est vers celle de points de vue féconds, me conduisant constamment à introduire, et à développer peu ou prou, des thèmes entièrement nouveaux que me porte mon génie particulier.
[· · ·] ces innombrables questions, notions, énoncés [· · ·] en naissent spontanément avec la force de l'évidence ; à la même façon qu'une lumière (même diffuse) qui surgit dans la nuit noire, semble faire naître du néant ces contours plus ou moins flous ou nets qu'elle nous révèle soudain."

("Récoltes et Semailles")

"Even more than towards the discovery of new questions, concepts and statements, my particular genius leads me towards fertile points of view, constantly prompting me to introduce and develop, to a greater or lesser extent, entirely new themes.
[· · ·] they spontaneously give birth to countless questions, concepts and statements, with the force of obviousness; in the same way that a light (even a diffuse one) which appears in the dark night seems to bring forth from nothingness these more or less blurred or sharp contours that it suddenly reveals to us."

- → New points of view are fertile.
- → They give birth to questions, concepts and statements just as a light which appears in dark night reveals to us contours from nothingness.

Points of view as languages

"Tout point de vue amène à développer un langage
qui l'exprime et lui est propre.
Avoir plusieurs "yeux" ou plusieurs "points de vue"
pour appréhender une situation, revient aussi (en mathématiques
tout au moins) à disposer de plusieurs langages différents pour la cerner."

("Récoltes et Semailles")

"Every point of view leads to the development of a language
that expresses it and is unique to it.
Having several "eyes" or several "points of view" to apprehend
a situation also means (in mathematics at least) having
several different languages to grasp it."

→ The development of new points of view,
which originates in listening
takes the form of language elaboration.

The necessity to multiply and combine points of view

"Comme son nom même le suggère, un "point de vue" en lui-même reste parcellaire. Il nous révèle un des aspects d'un paysage ou d'un panorama, parmi une multiplicité d'autres également valables, également "réels". C'est dans la mesure où se conjugent les points de vue complémentaires d'une même réalité, où se multiplient nos "yeux", que le regard pénètre plus avant dans la connaissance des choses."

("Récoltes et Semailles")

"As its very name suggests, a "point of view" in itself remains fragmentary. It reveals to us one aspect of a landscape or panorama, among a multitude of others that are equally valid, equally "real". It is to the extent that complementary points of view of the same reality combine, that our "eyes" multiply, that our gaze penetrates further into the knowledge of things."

- → The meaning of the expression "point of view" is literal for Grothendieck: it is a geographic point from which one looks at a landscape.
- → For Grothendieck, reality is a landscape.
  In particular, mathematical reality is a landscape.

Vision as unity of points of view

"Il arrive parfois qu'un faisceau de points de vue convergents
sur un même et vaste paysage, par la vertu de cela en nous apte à saisir
l'Un à travers le multiple, donne corps à une chose nouvelle ; à une chose
qui dépasse chacune des perspectives partielles, de la même façon
qu'un être vivant dépasse chacun de ses membres et de ses organes.
Cette chose nouvelle, on peut l'appeler une vision."

("Récoltes et Semailles")

"Sometimes, a bundle of converging points of view on the same vast landscape, by virtue of our ability to grasp the One through the many, gives shape to something new ; something that transcends each of the partial perspectives, in the same way that a living being transcends each of its limbs and organs.
This new thing can be called a vision."

→ Reality is unified but has to be seen from different points of view.
Unity is recovered when partial perspectives are transcended by a vision.

→ A vision is comparable to a living being.

Different levels of fertility

"La vision est aux points de vue dont elle paraît issue et qu'elle unit, comme la claire et chaude lumière du jour est aux différentes composantes du spectre solaire. Une vision vaste et profonde est comme une source inépuisable, faite pour inspirer et éclairer le travail non seulement de celui en qui elle est née un jour et qui s'est fait son serviteur, mais celui de générations, fascinées peut-être (comme il le fut lui-même) par ces lointaines limites qu'elle nous fait entrevoir."

("Récoltes et Semailles")

"The vision is to the points of view from which it appears to originate and which it unifies, as clear, warm daylight is to the different components of the solar spectrum. A broad and profund vision is like an inexhaustible source, designed to inspire and illuminate the work not only of the person in whom it was born one day and who made himself his servant, but also that of generations, perhaps fascinated (as he himself was) by the distant limits it allows us to glimpse."

- $\longrightarrow$ A vision can be experienced only by a living being.
- $\longrightarrow$ It has to be born in a living person who becomes its servant.
- $\longrightarrow$ It inspires and illuminates.
- $\longrightarrow$ It allows to glimpse at distant limits, as the horizon of a landscape.

The nature of Grothendieck's vision in mathematics

"Cette vaste vision unificatrice peut être décrite comme une géométrie nouvelle. [$\cdots$] On peut considérer que la géométrie nouvelle est, avant tout autre chose une synthèse entre ces deux mondes, jusque-là mitoyens et étroitement solidaires, mais pourtant séparés: le "monde arithmétique" dans lequel vivent les (soit-disant) "espaces" sans principe de continuité, et le monde de la grandeur continue [$\cdots$] Dans la vision nouvelle, ces deux mondes jadis séparés n'en forment plus qu'un seul."

("Récoltes et Semailles")

"This vast unifying vision can be described as a new geometry. [$\cdots$] We can consider that the new geometry is, above all else, a synthesis between these two worlds which until now have been contiguous and closely linked, yet separate: the "arithmetic" world, in which (so-called) "spaces" exist without any principle of continuity, and the world of continuous magnitude [$\cdots$]. In the new vision, these two formerly separate worlds now form a single whole."

$\longrightarrow$ The new unifying vision consists in a new geometry.

$\longrightarrow$ Above all else, it is a synthesis between two worlds which were formerly separate, the world of the discrete and the world of the continuous.

The first nucleus of Grothendieck's vision

"Le premier embryon de cette vision d'une "géométrie arithmétique" (comme je propose d'appeler cette géométrie nouvelle) se trouve dans les conjectures de Weil. $[\cdots]$

Sans elles, ni la cohomologie ℓ-adique, ni même le langage des topos n'auraient sans doute vu le jour. Pour mieux dire, cette "vaste vision unificatrice" de la géométrie (algébrique), de la topologie et de l'arithmétique que je me suis attaché à développer $[\cdots]$ c'est dans ces "conjectures de Weil" que j'en ai trouvé comme une première et saisissante ébauche."

("Récoltes et Semailles")

"The first nucleus of this vision of an "arithmetic geometry" (as I propose to call this new geometry) can be found in Weil's conjectures. $[\cdots]$

Without them, neither ℓ-adic cohomology nor even the language of toposes would probably have seen the light of the day. To put it more precisely, it was in these "Weil conjectures" that I found the first striking glimpse of the "vast unifying vision" of (algebraic) geometry, topology and arithmetic that I have endeavoured to develop."

$\longrightarrow$ Grothendieck's vision originated in the conjectures of André Weil (the brother of Simone Weil)!

The "Rosetta Stone" of mathematics

Weil's conjectures originated in the "Rosetta Stone" of mathematics,
as he explained in 1940 in a letter to his sister Simone Weil:

MYSTERIOUS DEEP ANALOGIES BETWEEN

Arithmetic	"Discrete" geometry of algebraic curves over finite fields	"Continuous" geometry of Riemann surfaces = algebraic curves over $\mathbb{C}$
$\mathbb{Z} \hookrightarrow \mathbb{Q}$ and more generally $O_E \hookrightarrow E$; O_E = ring of algebraic integers, E = number field	C = smooth projective curve over $\mathbb{F}_q$, F = field of rational functions on C	S = "Riemann surface" = smooth projective variety of dimension 1 over $\mathbb{C}$.
Riemann hypothesis for Zeta functions of $\mathbb{Q}$ or E	Analog of the Riemann hypothesis (proved by A. Weil)	
Structure of the Galois groups $\mathcal{G}al_{\mathbb{Q}}$ or $\mathcal{G}al_E$?	Structure of the Galois groups $\mathcal{G}al_F$?	Known structure of the Poincaré fundamental group of S

Deepening the analogy to the level of algebraic topology invariants

"Discrete" geometry of algebraic varieties over finite fields		"Continuous" geometry of algebraic varieties over $\mathbb{C} = \{$complex numbers$\}$
$X =$ smooth projective variety over $\mathbb{F}_q$		$X =$ smooth projective variety of dimension d over $\mathbb{C}$
Analog ?	$\Longleftarrow$	cohomology spaces H^i, $i \geq 0$, of the "continuous" topological space $X(\mathbb{C})$
?		finite dimensionality of every H^i + vanishing if $i > 2d$
?		Künneth formula for products $H^i(X \times X') = \cdots$
?		Poincaré duality between H^i and H^{2d-i}, $0 \leq i \leq 2d$.
?		Lefschetz' fixed points formula $\#\{$fixed points of $f : X \to X\}$ $= \sum_{0 \leq i \leq 2d} (-1)^i \cdot \mathrm{Tr}$ (linear action of f on H^i)

Frobenius canonical transform

$$(x_1, \cdots, x_n) \longmapsto (x_1^q, \cdots, x_n^q)$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

Far-reaching consequence: Weil conjectures

Metamorphosis of the notion of space through a new point of view

Classical topology of "continuous spaces"		Metamorphosis of topology as a new "science of toposes", encompassing all types of continuous and discrete spaces
topological space X	$\longmapsto$	$\mathcal{E}_X$ = topos of X = category of set-valued "sheaves" on X
continuous map $(X \xrightarrow{f} Y)$	$\longmapsto$	"topos morphism" $\mathcal{E}_X \to \mathcal{E}_Y$ consisting in $(\mathcal{E}_Y \xrightarrow{f^*} \mathcal{E}_X, \mathcal{E}_X \xrightarrow{f_*} \mathcal{E}_Y)$
		huge generalization to toposes constructed as $\widehat{\mathcal{C}}_J$ = category of set-valued "sheaves" on $\begin{cases} \mathcal{C} = \text{category,} \\ J = \text{"Grothendieck topology"} \\ \quad = \text{notion of covering in } \mathcal{C}\,. \end{cases}$
Poincaré fundamental groups = {homotopy classes of paths}		Galois-Poincaré groups of toposes, defined as automorphism groups of "fiber functors" of subtoposes of "étale covers"
singular cohomology		sheaf cohomology, consisting in "derived operations" (f^*, Rf_*) $(\overset{L}{\otimes}, R\mathcal{H}om)$ on "derived categories" of linear sheaves
finite dimensionality of H^i		"constructive coefficients", "local duality"
homology, Poincaré duality, Lefschetz' formula		"extraordinary operations" $(f_!, f^!)$, "global duality", "six operations formalism"

The geometrization of commutative algebra as scheme theory

The Serre construction:

- Start with a commutative algebra A which is finitely presentable over an algebraically closed field k and has no nilpotents.
- Associate to A its "maximal spectrum" $\mathrm{Spec}_m(A) = \{$maximal ideals of $A\}$ endowed with the "Zariski topology".
- Show that A induces on this topological space a "sheaf of commutative rings" $\mathcal{O}_A$ whose fibers are local commutative rings.
- Glue affine varieties $(\mathrm{Spec}_m(A), \mathcal{O}_A)$ to define algebraic varieties over k.

The Grothendieck generalization:

- Start with an arbitrary commutative ring A.
- Associate to A its "prime spectrum" $\mathrm{Spec}(A) = \{$prime ideals of $A\}$ endowed with the Zariski topology.
- Show that A induces on $\mathrm{Spec}(A)$ a sheaf of commutative rings $\mathcal{O}_A$ whose fibers are local commutative rings.
- Glue affine schemes $(\mathrm{Spec}(A), \mathcal{O}_A)$ to define schemes.

An alternate topossic presentation of the geometrization process

- Start with the algebraic theory $\mathbb{T}_{cr}$ of commutative rings.
- As $\mathbb{T}_{cr}$ is an algebraic theory, it is classified by the topos of presheaves
$$\mathcal{E}_{\mathbb{T}_{cr}} = \widehat{\mathcal{A}\!f\!f}$$
on $\quad \mathcal{A}\!f\!f = \{\text{category of finitely presentable commutative rings}\}^{\mathrm{op}}$
(later identified as the category of finitely presentable affine schemes)
endowed with the universal model
$$\begin{array}{ccc} \mathcal{A}\!f\!f^{\mathrm{op}} & \longrightarrow & \{\text{commutative rings}\}\,, \\ A & \longmapsto & A\,. \end{array}$$
- Consider the theory $\mathbb{T}_{\ell cr}$ of local commutative rings, which is a "quotient" of $\mathbb{T}_{cr}$.
- According to the PhD thesis of Monique Hakim (with Grothendieck), the theory $\mathbb{T}_{\ell cr}$ is classified by the subtopos
$$\mathcal{E}_{\mathbb{T}_{\ell cr}} = \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}} \hookrightarrow \widehat{\mathcal{A}\!f\!f} = \mathcal{E}_{\mathbb{T}_{cr}}$$
of sheaves for the Zariski topology,
endowed with the induced universal model
$$\mathcal{O} = \text{structure inner local ring in } \mathcal{E}_{\mathbb{T}_{\ell cr}}.$$
- The canonical functor
$$\ell : \mathcal{A}\!f\!f \longrightarrow \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}}$$
allows to restrict $\mathcal{O}$ to $\mathcal{O}_A$ on $\mathrm{Spec}(A)$ for any A in $\mathcal{A}\!f\!f$.
- Schemes (locally of finite presentation) are objects of $\mathcal{E}_{\mathbb{T}_{\ell cr}} = \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}}$ which are locally in the image of ℓ.

Generality of the Grothendieck geometrization process

Building on Grothendieck and his student Monique Hakim,
inheritors have shown that toposes allow to geometrize most of mathematics:

Theorem (Makkai, Reyes, Joyal, $\cdots$). –
Consider an arbitrary first-order theory $\mathbb{T}$, presented in "geometric" form.

(i) *For any topos $\mathcal{E}$, continuous families of structures of type $\mathbb{T}$ parametrized by $\mathcal{E}$ make up a category*
$$\mathcal{M}_{\mathbb{T}}(\mathcal{E})\,.$$

(ii) *Any topos morphism $\mathcal{E}' \xrightarrow{f} \mathcal{E}$ induces a "change of parameters" transform*
$$f^* : \mathcal{M}_{\mathbb{T}}(\mathcal{E}) \longrightarrow \mathcal{M}_{\mathbb{T}}(\mathcal{E}')\quad .$$

(iii) *Continuous families of structures of type $\mathbb{T}$ are classified by a (uniquely determined) topos $\mathcal{E}_{\mathbb{T}}$ in Grothendieck's sense, meaning that for any topos $\mathcal{E}$*
$$\mathcal{M}_{\mathbb{T}}(\mathcal{E}) = \{\text{continuous families of } \mathcal{E}\text{-parametrized } \mathbb{T}\text{-structures}\}$$
identifies with
$$\{\text{morphisms } \mathcal{E} \to \mathcal{E}_{\mathbb{T}}\} = \{\mathcal{E}\text{-parametrized mobile points of } \mathcal{E}_{\mathbb{T}}\}\,.$$

Theorem (Caramello). – *For any such theory $\mathbb{T}$, there is a one-to-one map*

$$\left\{\begin{array}{c}\text{"quotient" theories } \mathbb{T}' \text{ of } \mathbb{T} \\ \text{(defined by adding extra axioms)} \\ \text{considered up to provable equivalence}\end{array}\right\} \longrightarrow \{\text{subtoposes of } \mathcal{E}_{\mathbb{T}}\}\,,$$

$$\mathbb{T}' \longmapsto (\mathcal{E}_{\mathbb{T}'} \hookrightarrow \mathcal{E}_{\mathbb{T}}).$$

The "étale" topos as a subtopos

- We already considered $\mathbb{T}_{cr}$ = theory of commutative rings
  and its quotient $\mathbb{T}_{\ell cr}$ = theory of local commutative rings.
- A further quotient is
  $\mathbb{T}_{h\ell cr}$ = theory of strictly henselian local commutative rings.
- There is an induced chain of classifying toposes
  $$\mathcal{E}_{\mathbb{T}_{h\ell cr}} \hookrightarrow \mathcal{E}_{\mathbb{T}_{\ell cr}} = \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}} \hookrightarrow \mathcal{E}_{\mathbb{T}_{cr}} = \widehat{\mathcal{A}\!f\!f}\,.$$

Theorem (Hakim). –
The theory $\mathbb{T}_{h\ell cr}$ of strictly henselian local commutative rings is classified by the topos
$$\mathcal{E}_{\mathbb{T}_{h\ell cr}} = \widehat{\mathcal{A}\!f\!f}_{\acute{e}t}$$
of sheaves on the category $\mathcal{A}\!f\!f$ endowed with "étale" topology.

Fact:
The embedding functor $\{\text{schemes}\} = \mathrm{Sch} \hookrightarrow \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}}$
composed with
$$\mathcal{E}_{\mathbb{T}_{\ell cr}} = \widehat{\mathcal{A}\!f\!f}_{\mathrm{Zar}} \xrightarrow{\text{étale sheafification}} \widehat{\mathcal{A}\!f\!f}_{\text{ét}} = \mathcal{E}_{\mathbb{T}_{h\ell cr}}$$
allows to define cohomology of schemes with coefficients in
$$\mathbb{Q}_\ell = \{\text{field of } \ell\text{-adic numbers}\}, \quad \ell = \text{prime number},$$
so as to verify all demands of Weil
and even full six operations formalism.

A question about the general process of geometrization or topologization

Definition. – *A first-order (geometric) theory* $\mathbb{T}_p$ *is called "presheaf type"*
if it is classified by a topos of presheaves

$$\widehat{\mathcal{C}}$$

on some category $\mathcal{C}$.

Remarks:

(i) It has been proven by Caramello that if $\mathbb{T}_p$ is "presheaf type"

$$\mathcal{E}_{\mathbb{T}_p} = \widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}}$$

where $\mathcal{M}_{\mathbb{T}_p} = \{$category of finitely presentable set-valued structures of type $\mathbb{T}_p\}$.

(ii) Any first-order geometric theory $\mathbb{T}$ can be presented (in many ways) as a quotient of some presheaf-type theory $\mathbb{T}_p$.

Question: For a quotient $\mathbb{T}$ of a presheaf type theory $\mathbb{T}_p$ classified by a subtopos

$$\mathcal{E}_{\mathbb{T}} = \left(\widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}}\right)_{J_{\mathbb{T}}} \hookrightarrow \widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}} = \mathcal{E}_{\mathbb{T}_p}$$

necessarily defined by a topology $J_{\mathbb{T}}$ on $\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}$,
one can consider the associated geometrization of topologization functor

$$\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p} \hookrightarrow \widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}} \xrightarrow{J_{\mathbb{T}}\text{-sheafification}} \left(\widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}}\right)_{J_{\mathbb{T}}} = \mathcal{E}_{\mathbb{T}}\,.$$

When is this functor interesting?

Can "points of view" incarnate as points of a topos?

Grothendieck defined a multiplicity of cohomology theories

$$\begin{cases} - & \underline{\text{cohomology}} \text{ with } \underline{\text{coefficients in } \mathbb{Q}_\ell} \text{ for schemes over } \mathbb{Z}[\ell^{-1}], \\ - & \underline{\text{cohomology}} \text{ with } \underline{\text{coefficients in } \mathbb{Q}_p} \text{ for schemes over } k \supseteq \mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}, \\ - & \underline{\text{De Rham cohomology}} \text{ for schemes over } k \supseteq \mathbb{Q}, \end{cases}$$

in addition to previously defined

$$\begin{cases} - & \underline{\text{singular Betty cohomology}} \text{ for algebraic varieties over } \mathbb{C}, \\ - & \underline{\text{Hodge cohomology}} \text{ for algebraic varieties over } \mathbb{C}. \end{cases}$$

Motivic conjecture (in a form proposed by Caramello). –

Consider a "good" geometric category $\mathcal{G}$. Then $\mathcal{G}$ should define

$$\begin{cases} (1) & \text{a } \underline{\text{presheaf type theory}}\ \mathbb{T}_p\ \underline{\text{classified}} \text{ by the } \underline{\text{topos}} \\ & \mathcal{E}_{\mathbb{T}_p} = \widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}} \quad \text{where } \mathcal{M}_{\mathbb{T}_p} = \{\text{finitely presentable structures of type } \mathbb{T}_p\}, \\ (2) & \text{the } \underline{\text{quotient theory}}\ \mathbb{T} \text{ of } \mathbb{T}_p\ \underline{\text{classified}} \text{ by the } \underline{\text{subtopos}} \\ & \mathcal{E}_{\mathbb{T}} = \big(\widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}}\big)_{J_{at}} \hookrightarrow \widehat{\mathcal{M}^{\mathrm{op}}_{\mathbb{T}_p}} \\ & \text{defined by the "}\underline{\text{atomic topology}}\text{" (for which } \underline{\text{any morphism}}\ X' \to X \text{ is a } \underline{\text{covering}} \text{ of } X), \end{cases}$$

so that

$$\begin{cases} \text{(i)} & \text{the } \underline{\text{points}} \text{ of } \mathcal{E}_{\mathbb{T}} \text{ would correspond to} \\ & \text{the } \underline{\text{cohomology functors}} \text{ on } \mathcal{G} \text{ which satisfy } \underline{\text{Weil's demands}}, \\ \text{(ii)} & \text{the } \underline{\text{theory}}\ \mathbb{T} \text{ would be automatically "}\underline{\text{complete}}\text{"}, \\ & \text{implying } \underline{\text{all cohomology functors}} \text{ would share the } \underline{\text{same properties}}. \end{cases}$$